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STUDY OF MAXIMUM STRESS FIELD ALONGSIDE CRACKS EMERGING FROM CONTOURS OF OPENINGS IN A PERFORATED PLATE

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A considerable number of papers have appeared in recent years (see the reviews [1, 2]) in which the stress state alongside cracks emerging from the contour of a single opening was investigated. The analogous problem of the stretching of a plate with a single opening was the subject of [3].

§1. Let there be a doubly periodic array of circular openings having a radius λ ($\lambda < 1$) and centers at the points

$$P_{mn} = m\omega_1 + n\omega_2 \ (m,\ n = 0,\ \pm 1,\ \pm 2, \ldots),\ \omega_1 = 2,\ \omega_2 = 2le^{i\alpha},\ l > 0,\ \operatorname{Im}\omega_2 > 0.$$

Symmetric linear slits originate from the contours of the openings (Fig. 1). The contours of the circular openings and the edges of the slits are free of loads. We consider the problem of the stretching of such a perforated plate by constant forces $\sigma_2 = \sigma_y^\infty$ in a direction perpendicular to the line of the slits. Because of the symmetry of the boundary conditions and the geometry of the region D occupied by the plate material, the stresses are doubly periodic functions with fundamental periods ω_1 and ω_2.

To solve the problem in reasonable fashion, we combine the method developed for the solution of a doubly periodic elastic problem [4] with the method for plotting [5, 6] in explicit form the Kolosov–Muskhelishvili potentials corresponding to unknown normal displacements along the slits.

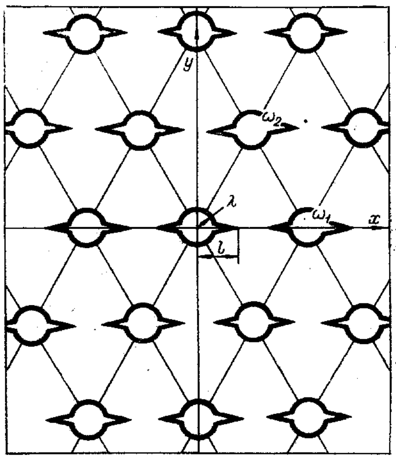

Fig. 1

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We represent the stresses and displacements [7] through the Kolosov–Muskhelishvili potentials $\Phi(z)$ and $\Psi(z)$

$$\sigma_x + \sigma_y = 4\,\mathrm{Re}\,\Phi(z)\ (z = x + iy),\tag{1.1}$$
$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)],$$
$$2\mu(u + iv) = \varkappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)},$$
$$\Phi(z) = \varphi'(z),\ \Psi(z) = \psi'(z),$$

$$\varkappa = \begin{cases} 3 - 4\nu & \text{(two-dimensional deformation)},\\ (3 - \nu)/(1 + \nu) & \text{(two-dimensional stress state)},\end{cases}$$

μ and ν are the shear modulus and Poisson coefficient, respectively.

Based on Eqs. (1.1) and the boundary conditions at the contours of the circular openings and at the edges of the slits, the problem reduces to a determination of the two functions $\Phi(z)$ and $\Psi(z)$, which are analytic in the region D, from the boundary conditions

$$\Phi(\tau) + \overline{\Phi(\tau)} - [\bar{\tau}\Phi'(\tau) + \Psi(\tau)]e^{2i\theta} = 0;\tag{1.2}$$
$$\Phi(t) + \overline{\Phi(t)} + t\overline{\Phi'(t)} + \overline{\Psi(t)} = 0,\tag{1.3}$$

where $\tau = \lambda e^{i\theta} + m\omega_1 + n\omega_2$, m, n = 0, ± 1, ± 2 ...; t is the affix of points on the edges of the slits.

We seek a solution of the boundary-value problem (1.2), (1.3) in the form

$$\Phi(z) = \Phi_1(z) + \Phi_2(z),\ \Psi(z) = \Psi_1(z) + \Psi_2(z);\tag{1.4}$$
$$\Phi_1(z) = \frac{1}{2\pi}\int_L g(x)\,\zeta(x - z)\,dx + A,\tag{1.5}$$
$$\Psi_1(z) = \frac{1}{2\pi}\int_L [\zeta(x - z) + Q(x - z) - x\gamma(x - z)]\,g(x)\,dx + B;$$

$$\Phi_2(z) = \frac{1}{4}\sigma_y^\infty + \sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}\gamma^{(2k)}(z)}{(2k+1)!},$$
$$\Psi_2(z) = \frac{1}{2}\sigma_y^\infty + \sum_{k=0}^{\infty}\beta_{2k+2}\frac{\lambda^{2k+2}\gamma^{(2k)}(z)}{(2k+1)!} - \sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}Q^{(2k+1)}(z)}{(2k+1)!},\tag{1.6}$$
$$\mathrm{Im}\,\alpha_{2k} = \mathrm{Im}\,\beta_{2k} = 0,$$

where the integrals in (1.5) are taken along the line $L = \{[-l, -\lambda] + [\lambda, l]\}$; $\gamma(z)$ and $\zeta(z)$ are Weierstrass functions; Q(z) is a special meromorphic function [4]; g(x) is the function sought; A and B are constants.

One should add to Eqs. (1.4)–(1.6) the additional condition resulting from the physical essence of the problem

$$\int_L g(x)\,dx = 0.\tag{1.7}$$

At congruent points, the functions $\gamma(z)$, $\zeta(z)$, and Q(z) satisfy the conditions [4]

$$\gamma(z + \omega_j) - \gamma(z) = 0,\ \zeta(z + \omega_j) - \zeta(z) = \delta_j\ (j = 1,\ 2),\tag{1.8}$$
$$Q(z + \omega_j) - Q(z) = \overline{\omega_j}\gamma(z) + \gamma_j,$$
$$\delta_j = 2\zeta(\omega_j/2),\ \gamma_j = 2Q(\omega_j/2) - \overline{\omega_j}\gamma(\omega_j/2),$$
$$\delta_1\omega_2 - \delta_2\omega_1 = 2\pi i,\ \gamma_2\omega_1 - \gamma_1\omega_2 = \delta_1\overline{\omega_2} - \delta_2\overline{\omega_1}.$$

A zero value for the principal vector of the forces acting on an arc connecting two congruent points in D is equivalent to

$$q(z + \omega_j) - q(z) = 0\ (j = 1,\ 2),\quad q(z) = \varphi(z) + z\overline{\Phi(z)} + \overline{\psi(z)}$$

and leads to the relation

$$A + \bar{A} + \bar{B} = -(1/\omega_1)\{\delta_1 a + \bar{\gamma}_1 a + \bar{\delta}_1(a + \bar{a}) - \alpha_2\lambda^2(\delta_1 + \bar{\gamma}_1) - \beta_2\lambda^2\bar{\delta}_1\}$$

when Eqs. (1.7), (1.8) are considered.

One can verify that the functions (1.4)-(1.6) under the conditions (1.7) define a class of symmetric problems with doubly periodic distribution of stresses.

The unknown function g(x) and the constants α_{2k+2} and β_{2k+2} must be determined from the boundary conditions (1.2) and (1.3).

Because the condition of double periodicity is satisfied, the system of boundary conditions (1.2) is replaced by a single functional equation on the contour $\tau = \lambda e^{i\theta}$, for example, and the system of conditions (1.3) by a boundary condition on L.

To formulate equations with respect to the coefficients α_{2k+2} and β_{2k+2} of the functions $\Phi_2(z)$ and $\Psi_2(z)$, we represent the boundary condition (1.2) in the form

$$\Phi_2(\tau) + \overline{\Phi_2(\tau)} - [\bar{\tau}\Phi_2'(\tau) + \Psi_2(\tau)]e^{2i\theta} = f_1(\theta) + if_2(\theta), \tag{1.9}$$

where

$$f_1(\theta) + if_2(\theta) = -\Phi_1(\tau) - \overline{\Phi_1(\tau)} + [\bar{\tau}\Phi_1'(\tau) + \Psi_1(\tau)]e^{2i\theta}.$$

We assume that the function $f_1(\theta) + if_2(\theta)$ is expandable into a Fourier series on $|\tau| = \lambda$. Because of symmetry, this series has the form

$$f_1(\theta) + if_2(\theta) = \sum_{k=-\infty}^{\infty} A_{2k}e^{2ik\theta}, \quad \operatorname{Im} A_{2k} = 0; \tag{1.10}$$

$$A_{2k} = -\frac{1}{2\pi}\int_L g(x) f_{2k}(x)\,dx, \tag{1.11}$$

$$f_{2k}(x) = \frac{1}{2\pi}\int_0^{2\pi} \{2\operatorname{Re}[A + \zeta(x - \lambda e^{i\theta})]\} e^{-2ik\theta} d\theta - \frac{1}{2\pi}\int_0^{2\pi} \{\lambda e^{-i\theta}\gamma(x - \lambda e^{i\theta}) +$$
$$+ \zeta(x - \lambda e^{i\theta}) + Q(x - \lambda e^{i\theta}) - x\gamma(x - \lambda e^{i\theta}) + B\} e^{-i(2k-2)\theta} d\theta.$$

Because of the cumbersome nature of the functions $f_{2k}(x)$, the result of the integration, which was obtained by means of the theory of residues, is not presented.

Replacing $\Phi_2(\tau)$, $\overline{\Phi_2(\tau)}$, $\Phi'_2(\tau)$, and $\Psi_2(\tau)$ on the left side of the boundary condition (1.9) by their expansions in Laurent series in the neighborhood of $z = 0$ and the right side of Eq. (1.9) by the Fourier series (1.10), and equating coefficients of identical powers of $e^{i\theta}$, we obtain [4] two infinite systems of algebraic equations with respect to the coefficients α_{2k+2} and β_{2k+2}. They are not given because of their extremely unwieldy form (see the systems (3.3) and (3.5) in Chap. 1 of [4]).

Requiring that the functions (1.4) satisfy the boundary condition on the slit edge L, we obtain the singular integral equation

$$\frac{1}{2\pi}\int_L g(t) K(t - x)\,dt + H(x) = 0, \quad K(x) = 3\zeta(x) + Q(x) - x\gamma(x),$$
$$H(x) = A + \bar{A} + \bar{B} + 2\Phi_2(x) + x\Phi_2'(x) + \Psi_2(x). \tag{1.12}$$

The singular equation (1.12) and the systems (3.3) and (3.5) from [4] are basic equations of the problem which make it possible to determine the function g(x) and the coefficients α_{2k+2} and β_{2k+2}. Knowing the functions g(x), $\Phi_2(z)$, and $\Psi_2(z)$, one can determine the stress–strain state of a perforated plate. In the mechanics of brittle fracture [8], there is particular interest in the coefficient of stress intensity in the neighborhood of the end of a crack. In the present case, a crack with one end at $x = \lambda$ emerges from the surface of a circular opening which is free of external forces. In this case the stress is bounded at the end $x = \lambda$ and has a singularity at the other end at $x = l$. In particular, we have for the coefficient of stress intensity K_I at the ends of the crack at $x = \pm l$:

$$K_I = 2\sqrt{2\pi|x - l|}\,g(x).$$

The function g(x) is bounded in the neighborhood of $x = \pm\lambda$ and has a singularity of order 1/2 in the neighborhood of $x = \pm l$.

The development of a crack is determined by some supplementary condition assigned at the tip of the crack. For a linearly elastic body, the supplementary condition is the Griffith–Irwin local fracture criterion

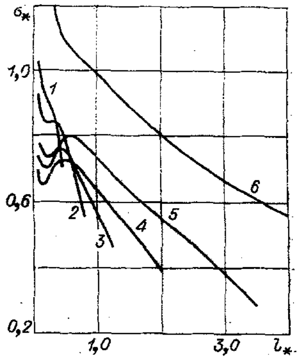

Fig. 2

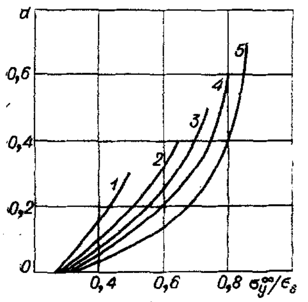

Fig. 3

$K_I = K_{Ic}$ (K_{Ic} is a constant which characterizes the resistance of a material to crack propagation). This condition makes it possible to determine the value of the maximum (critical) external forces σ_y^∞.

Using the expansions [4] in the basic parallelogram of the periods

$$\zeta(z) = \frac{1}{z} - \sum_{j=1}^{\infty} \frac{g_{j+1} z^{2j+1}}{2^{2j+2}}, \quad g_k = \sum_{m,n}{}' \frac{1}{T^{2k}},$$

$$\gamma(z) = \frac{1}{z^2} + \sum_{j=1}^{\infty} \frac{(2j+1)\, g_{j+1}}{2^{2j+2}} z^{2j}, \quad \rho_k = \sum_{m,n}{}' \frac{\overline{T}}{T^{2k+1}},$$

$$Q(z) = \sum_{j=1}^{\infty} \frac{(2j+2)\,\rho_{j+1}}{2^{2j+2}} z^{2j+1}, \quad T = \frac{1}{2} P_{mn},$$

$$m,\ n = 0,\ \pm 1,\ \pm 2, \ldots;\ k = 2,\ 3, \ldots,$$

we bring Eq. (1.12) after some simple transformations to the form

$$\frac{1}{\pi} \int_L \frac{p(\xi)}{\xi - \xi_0} d\xi + \frac{1}{\pi} \int_L p(\xi) K_0(\xi - \xi_0)\, d\xi + H(\xi_0) = 0, \tag{1.13}$$

$$p(\xi) = g(t), \quad \xi = \frac{t}{l}, \quad \xi_0 = \frac{x}{l}, \quad \lambda_1 = \frac{\lambda}{l}, \quad L = \{[-1, -\lambda_1] + [\lambda_1, 1]\},$$

$$K_0(\xi) = K_*(\xi) - K(\xi),$$

$$K(\xi) = \sum_{j=0}^{\infty} K_j \left(\frac{l}{2}\right)^{2j+2} \xi^{2j+1}, \quad \lambda \leqslant l < 1,$$

$$K_*(\xi) = \sum_{j=0}^{\infty} K_j^* \left(\frac{l}{2}\right)^{2j+2} \xi^{2j+1}, \quad K_0 = \omega \operatorname{Re} \delta_1,$$

$$K_j = g_{j+1}, \quad K_0^* = -\frac{\omega_1}{2}(\overline{\gamma}_1 + \overline{\delta}_1), \quad K_j^* = (j+1)(\rho_{j+1} - g_{j+1}), \quad j = 1, 2,$$

$$H(\xi_0) = \sigma_y^\infty + \frac{1}{\omega_1}\left[\alpha_2 \lambda^2 (\delta_1 + \overline{\gamma}_1) + \beta_2 \lambda^2 \overline{\delta}_1\right] + 2\Phi_2(\xi_0 l) + \xi_0 l \Phi_2'(\xi_0 l) + \Psi_2(\xi_0 l).$$

One should add to the singular integral equation the supplementary condition (1.7) converted to the form

$$\int_L p(\xi)\, d\xi = 0. \tag{1.14}$$

Condition (1.14) determines the symmetric solution of the problem. For $p(\xi) = p(-\xi)$, Eq. (1.13) takes the form

$$\frac{2}{\pi} \int_{\lambda_1}^{1} \frac{\xi p(\xi)\, d\xi}{\xi^2 - \xi_0^2} + \frac{1}{\pi} \int_{\lambda_1}^{1} K_0^*(\xi, \xi_0)\, p(\xi)\, d\xi + H(\xi_0) = 0, \tag{1.15}$$

$$K_0^*(\xi, \xi_0) = K_0(\xi - \xi_0) + K_0(\xi + \xi_0), \quad \lambda_1 \leqslant \xi_0 < 1.$$

We convert Eq. (1.15) to a form more suitable for the determination of its approximate solution. For this purpose, we make the substitution of variables

$$\xi^2 = u = \frac{1-\lambda_1^2}{2}(\tau+1)+\lambda_1^2, \tag{1.16}$$

$$\xi_0^2 = u_0 = \frac{1-\lambda_1^2}{2}(\eta+1)+\lambda_1^2.$$

Then the integration segment $[\lambda_1, 1]$ transforms into the segment $[-1, 1]$ and the converted equation (1.15) takes the form

$$\frac{1}{\pi}\int_{-1}^{1}\frac{p(\tau)\,d\tau}{\tau-\eta}+\frac{1}{\pi}\int_{-1}^{1}p(\tau)\,B(\eta,\tau)\,d\tau+H_{**}(\eta)=0, \tag{1.17}$$

where

$$p(\tau) = p(\xi);$$

$$B(\eta,\tau) = \frac{1-\lambda_1^2}{2}\sum_{j=0}^{\infty}(K_j^*-K_j)\left(\frac{l}{2}\right)^{2j+2}u_0^j A_j;$$

$$A_j = \left\{(2j+1)+\frac{(2j+1)(2j)(2j-1)}{1\cdot 2\cdot 3}\left(\frac{u}{u_0}\right)+\ldots+\frac{(2j+1)(2j)(2j-1)\ldots[(2j+1)-(2j+1-1)]}{1\cdot 2\ldots(2j+1)}\left(\frac{u}{u_0}\right)^j\right\}.$$

For simplicity we set $H_{**}(\eta) = H_*(\xi_0)$. Remember that the function $H_{**}(\eta)$ contains the unknown coefficients α_{2k+2}, and β_{2k+2}.

We seek a solution of Eq. (1.17) which is bounded on the left end. The singular integral equation is usually regularized in accordance with Karleman–Vecu by reduction to a Fredholm equation. However, in the solution of problems which are of interest for applications, it is convenient to use one of the methods for direct solution of singular equations [9, 10]. We use the method developed in [11]. We represent the solution in the form

$$p(\tau) = p_0(\tau)\sqrt{(1+\tau)/(1-\tau)}. \tag{1.18}$$

Here $p_0(\tau)$ is Hölder-continuous on $[-1, 1]$ with the function $p_0(\tau)$ being replaced by Lagrangian interpolation polynomials constructed at Chebyshev mesh points,

$$L_n[p_0,\tau] = \frac{1}{n}\sum_{k=1}^{n}(-1)^{k+1}p_k^0\frac{\cos n\theta\sin\theta_k}{\cos\theta-\cos\theta_k}, \quad \tau=\cos\theta,$$

$$p_k^0 = p_0(\tau_k), \quad \tau_m=\cos\theta_m, \quad \theta_m=\frac{2m-1}{2n}\pi, \quad m=1,2,\ldots,n.$$

Using the relation [10]

$$\frac{1}{\pi}\int_0^{\pi}\frac{\cos n\tau\,d\tau}{\cos\tau-\cos\theta}=\frac{\sin n\theta}{\sin\theta}, \quad 0\leqslant\theta\leqslant\pi, \; n=1,\,2,\ldots,$$

$$\int_{-1}^{1}\frac{F(x)\,dx}{\sqrt{1-x^2}}=\frac{\pi}{n}\sum_{\nu=1}^{n}F(\cos\theta_\nu)$$

and Eqs. (1.11) and (1.16), we obtain the quadrature formulas

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{p(\tau)\,d\tau}{\tau-\eta}=\frac{1+\cos\theta}{n\sin\theta}\sum_{\nu=1}^{n}p_\nu^0\sum_{m=0}^{n=1}\cos m\theta_\nu\sin m\theta+\frac{1}{2n}\sum_{\nu=1}^{n}p_\nu^0, \tag{1.19}$$

$$\frac{1}{2\pi}\int_{-1}^{1}p(\tau)\,B(\eta,\tau)\,d\tau=\frac{1}{2n}\sum_{\nu=1}^{n}(1+\cos\theta_\nu)\,B(\cos\theta,\cos\theta_\nu)\,p_\nu^0;$$

$$A_{2k}=-\frac{1}{2n}\sum_{\nu=1}^{n}p_\nu^0(1+\cos\theta_\nu)\,f_{2k}^*(\cos\theta_\nu), \tag{1.20}$$

where

$$f_{2k}^*(\tau)=\frac{1-\lambda_1^2}{2}f_{2k}^*(\xi^2); \quad \xi l f_{2k}^*(\xi^2)=f_{2k}(t).$$

Equations (1.19) and (1.20) make it possible to replace the basic equations by an infinite system of linear algebraic equations with respect to the approximate values p_ν^0 of the desired function at mesh points and also with respect to the coefficients α_{2k+2} and β_{2k+2}.

After simple computations, the singular equation is replaced by the system

$$\sum_{\nu=1}^{n} a_{m\nu} p_\nu^0 + \frac{1}{2} H_{**}(\eta_m) = 0, \quad m = 1, 2, \ldots, n, \tag{1.21}$$

$$a_{m\nu} = \frac{1}{2n}\left[1 + \operatorname{ctg}\frac{\theta_m}{2}\operatorname{ctg}\frac{\theta_m + (-1)^{|m-\nu|}\theta_\nu}{2} + (1+\tau_\nu) B(\eta_m, \tau_\nu)\right], \quad \tau_m = \eta_m.$$

The system (1.21) is connected (closed) by two infinite systems ((3.3) and (3.5) from [4]) in which the expression (1.20) is inserted in place of A_{2k}. The three systems specified completely determine the solution of the problem. The coefficient of stress intensity is given by the expression

$$K_I = \frac{2}{n}\sqrt{\pi l (1 - \lambda_1^2)} \sum_{\nu=1}^{n} (-1)^\nu p_\nu^0 \operatorname{ctg}\frac{\theta_\nu}{2}.$$

A regular triangular array with $\omega_1 = 2$ and $\omega_2 = 2e^{(1/3)i\pi}$ was used for the numerical calculations. The calculations were performed on an M-222 computer. In the system (1.21) we set n = 10, 20, and 30, which corresponded to subdivision of the interval into 10, 20, and 30 Chebyshev mesh points, respectively. Using one of them, the unknown coefficients β_{2k+2} were then eliminated from the remaining equations. It turned out that the values of the critical external load and also of the coefficients α_{2k+2} and β_{2k+2} were essentially unchanged beginning with n = 20 (agreed up to the sixth digit).

Figure 2 shows calculated results for the critical (maximum) load $\sigma_* = \sigma_y^\infty \sqrt{\omega_1}/K_{Ic}$ as a function of the crack length $l_* = (l - \lambda)/\lambda$ for several values of the opening radius λ = 0.6, 0.5, 0.4, 0.3, 0.2, and 0.1 (curves 1-6).

As is apparent, stable development of a system of cracks (their mutual reinforcement) is possible for a regular triangular array at certain values of the radius λ of a circular opening. For a plate with a double periodic system of cracks ($\lambda = 0$) having the same basic periods, there is no possibility of stabilization of crack growth.

A solution for other external loads can also be obtained in a similar manner.

§2. Now let the material of a perforated plate be ideally elastoplastic obeying the Tresca–Saint Venant condition, according to which the maximum tangential stress at each point of a body does not exceed the shear flow limit τ_s ($2\tau_s = \sigma_s$, where σ_s is the limit of stretch flow). It is known from the elastic solution of the problem of stretch in a perforated plate that the maximum stresses σ_y occur at the points $t = \pm\lambda + m\omega_1 + n\omega_2$ (m, n = 0, ± 1, ± 2,...). Regions of plastic deformation will arise under a certain load in this case.

We consider the problem of initial development of plastic deformations under uniaxial stretching of a thin perforated plate by forces σ_y^∞. We assume that the plastic deformations are concentrated along certain slip lines originating at the contour of an opening. The general tendency toward the formation of plastic regions in the first stages of their development in the form of narrow slip bands occupying an insignificant portion of the body in comparison with its elastic portion is well known experimentally [12, 13]. This is particularly typical of materials which have a definite small area of flow (metals such as soft steel which tend toward retardation of flow and which are usually better described by the Tresca–Saint Venant conditions) and also in the presence of a stress state with sufficiently high stress gradients. From exact calculations, plastic regions have a tendency toward localization in the slip line [14, 15]. For example, from an exact solution of the elastoplastic problem of biaxial stretching of a plate with circular openings found in [15], the plastic zone is transformed from a circular region to an elongated region with a width-to-length ratio of approximately 1:4 even for deviation of the stress state at infinity from the uniform state by 0.1 ($\Delta\sigma/\sigma \simeq 0.1$). As shown by experiment, plastic regions in such cases will be a segment of length d (d = $l - \lambda$) (see Fig. 1). The thickness of the zone can be assumed to be zero. In thin plates, it can be realized physically in the form of a slip plane at 45° to the plane of the plate. Because of localization of plastic deformations, the elastoplastic problem being considered can be reduced to the boundary problem in the two-dimensional theory of elasticity discussed in Sec. 1 with σ_s replacing the right side of the boundary condition (1.3). The quantity l, which now characterizes the length of the plasticity zone, appears in the solution of Eq. (1.13) as an unknown parameter subject to determination.

Since the stress in an ideal elastoplastic material is bounded, the solution of the singular integral equation (1.13) should be sought in the class of everywhere bounded functions (stresses). The boundedness of the stresses at the ends $\pm l$ serves to determine the parameter l from which one can determine the length of the plastic zone.

This means that for solution of Eq. (1.13) in the class (1.18) in conjunction with the two infinite systems (see systems (3.3) and (3.5) in [4]), the equation

$$\sum_{\nu=1}^{n} (-1)^{\nu} p_{\nu}^{0} \operatorname{ctg} \frac{\theta_{\nu}}{2} = 0 \tag{2.1}$$

should be added to the system (1.21).

Equation (2.1) together with the systems noted makes up a closed system for determination of all unknowns in the problem. However, solution of this closed system for a given load σ_y^{∞} is difficult because of the nonlinearity of the algebraic equations with respect to the unknown parameter l. It is therefore simpler to assume a given value of l and to determine the load acting on the plate.

Figure 3 presents curves for the dependence of the length of the plasticity zone on the dimensionless external load $\sigma_y^{\infty}/\sigma_s$ for several values of the opening radius $\lambda = 0.6$, 0.5, 0.4, 0.3, and 0.2 (curves 1-5).

Note that when $|\omega_2| \to \infty$, we have a periodic system of circular openings with slits located along the x axis; with $|\omega_1| \to \infty$ and ω_2 finite, we obtain a plate with a periodic system of parallel circular openings with slits.

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